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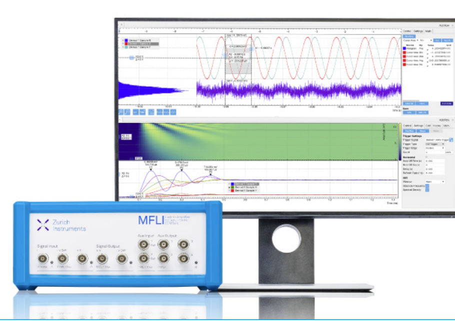

Light Propagation in Anisotropic Disordered Media

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Abstract. We present a semi-analytical theory for light propagation in three dimensional, strongly scattering, disordered, anisotropic dielectrics. The anisotropy of the system is incorporated by a tensor dielectric function. By starting at Maxwell's equations, we derive a general transport theory for light including transport quantities such as energy transport velocity, transport mean free path and diffusion coefficient. This approach is based on a fully vectorial treatment of the generalized kinetic equation and also incorporated a generalized Ward identity for these systems. Furthermore, self-interference contributions to the transport are included by means of a generalized localization theory based on a cooperon resummation first derived for electrons by Vollhardt and Wölfle [1]. Numerical evaluations will be presented.

Keywords: Light Propagation, Disordered Media, Light Localization, Anisotropic Media, Light Transport, Diffusion

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INTRODUCTION

Anisotropy and its effects on the propagation and the transport of electromagnetic waves in complex dielectrics remains to be a very exciting and intensly studied subject [2, 3, 4, 5]. Despite its rather great importance on experiments and applications, such as diffusing optical tomography, the theoretical description often employs a large variety of approximations such as a reduction to scalar wave equations [5]. To overcome the limitations of unnecessary approximations, we present here a theory of electromagnetic wave diffusion in disordered anisotropic dielectrics, based on a fully vectorial treatment. Therefore, our approach also includes polarization effects and generalized existing theories of light diffusion in disordered media e.g. in [6] and light localization effects as discussed in ref. [7].

MODEL

We consider light propagation according to the wave equation for the electric field $\vec{E}$ as derived from Maxwell's equations and Fourier transformed from time t to light frequnecy ω

$$-\vec{\nabla}\times\vec{\nabla}\times\vec{E}_{\omega}(\vec{r})+\omega^2\boldsymbol{\varepsilon}(\vec{r})\vec{E}_{\omega}(\vec{r})=\vec{J}_{\omega}(\vec{r}). \qquad (1)$$

Here, $\vec{J}_{\omega}(\vec{r})$ represents a general source of the light field due to an external current $\vec{j}$ as $\vec{J}_{\omega}(\vec{r})=-i\omega\vec{j}_{\omega}(\vec{r})$. The dielectric function $\boldsymbol{\varepsilon}(\vec{r})$ has a tensor form, denoted by bold face characters. It is given by

$$\boldsymbol{\varepsilon}(\vec{r})=\boldsymbol{\varepsilon}_b+(\boldsymbol{\varepsilon}_s-\boldsymbol{\varepsilon}_b)V(\vec{r}) \qquad (2)$$

and describes the disordered medium by means of the homogeneous but anisotropic dielectric constant of the background medium $\boldsymbol{\varepsilon}_b$ and the corresponding and also anisotropic dielectric constant of the scatterers $\boldsymbol{\varepsilon}_s$ through the position dependent function $V(\vec{r})=\sum_{\vec{R}}S_{\vec{R}}(\vec{r}-\vec{R})$ which consists of a set of localized shape functions $S_{\vec{R}}(\vec{r})$ of the individual scatterers at random locations $\vec{R}$ within the disordered system.

For a given particular disorder realization $\boldsymbol{\varepsilon}(\vec{r})$ the solution of the wave equation, Eq. (1), is given in terms of the Green's tensor $\boldsymbol{G}^{\omega}(\vec{r},\vec{r}\,')$ according to

$$\vec{E}_{\omega}(\vec{r})=\int \mathrm{d}^3r'\boldsymbol{G}^{\omega}(\vec{r},\vec{r}\,')\vec{J}_{\omega}(\vec{r}\,'). \qquad (3)$$

The disorder averaged Green's tensor regains translational invariance $\boldsymbol{G}^{\omega}(\vec{r})=\langle\boldsymbol{G}^{\omega}(\vec{r},\vec{r}\,')\rangle$ and its Fourier transform $\boldsymbol{G}^{\omega}_{\vec{q}}$ may be shown to obey

$$\boldsymbol{G}^{\omega}_{\vec{q}}=\left(\boldsymbol{G}_0^{-1}(\vec{q},\omega)-\boldsymbol{\Sigma}^{\omega}_{\vec{q}}\right)^{-1} \qquad (4)$$

where self-energy $\boldsymbol{\Sigma}^{\omega}_{\vec{q}}$ arises from scattering processes off the "scattering potential" $(\boldsymbol{\varepsilon}_s-\boldsymbol{\varepsilon}_b)V(\vec{r})$, see Eq. (2). The use of disorder averaged quantities is necessary in order to compare to experimentally measured data.

DISCUSSION

Due to the translational invariance of the disorder averaged quantities such as the Green's tensor, a transport theory ought to be formulated in terms of correlation functions and not in terms of single particle quantities. Therefore, we define the field correlation tensor (of

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fourth rank) $\mathbf{\Phi}_{\omega_1\,\omega_2}(\vec{r}_1,\vec{r}_2,\vec{r}_3,\vec{r}_4)$ as the disorder averaged product of non-averaged single particle Green's tensors as

$$\mathbf{\Phi}_{\omega_1\,\omega_2}(\vec{r}_1,\vec{r}_2,\vec{r}_3,\vec{r}_4) = \langle \boldsymbol{G}^{\omega_1}(\vec{r}_1,\vec{r}_3)\otimes \boldsymbol{G}^{\omega_2}(\vec{r}_2,\vec{r}_4)\rangle \qquad (5)$$

where the notation $\ldots\otimes\ldots$ refers to the tensor product of two tensors of second rank, operating in the retarded and advanced subspace of the propagators, repectively.

The Fourier transformed correlation tensor $\mathbf{\Phi}^{\omega}_{\vec{q}_1,\vec{q}_2}(\vec{Q},\Omega)$ may be shown to obey the so-called Bethe-Salpter equation

$$\mathbf{\Phi}^{\omega}_{\vec{q},\vec{q}\,'}(\vec{Q},\Omega) = \boldsymbol{G}^{\omega_+}_{\vec{q}_+}\otimes\left(\boldsymbol{G}^{\omega_-}_{\vec{q}_-}\right)^{*}\left[\delta(\vec{q}-\vec{q}\,')\ \ \otimes\right.$$
$$\left. + \int\frac{\mathrm{d}q''}{(2\pi)^3}\boldsymbol{\gamma}^{\omega}_{\vec{q},\vec{q}\,''}(\vec{Q},\Omega)\mathbf{\Phi}^{\omega}_{\vec{q}\,'',\vec{q}\,'}(\vec{Q},\Omega)\right]. \qquad (6)$$

In order to derive the Bethe-Salpter eqution, Eq. (6), we introduced center of mass and relative frequencies Ω, ω as well as momenta $\vec{Q}$, $\vec{q}$, respectively, according to $\omega_{1,2}=\omega\pm\Omega/2\equiv\omega_{\pm}$, $\vec{q}_{1,2}=\vec{q}\pm\Omega/2\equiv\vec{q}_{\pm}$ and $\vec{q}_{3,4}=\vec{q}\,'\pm\Omega/2\equiv\vec{q}\,'_{\pm}$. The center of mass coordinates $(\vec{Q},\Omega)$ are associated with the time and position dependance of the diffusing modes of the energy density within the system, whereas the relative coordinates are associated with the single particle quantities, i.e. the frequency of the light field and its wave vector.

From the Bethe-Salpeter equation, Eq. (6), we derive the so-called kinetic equation, which is analogous to the Boltzmann equation and given by

$$\left(-\Delta\boldsymbol{g}_{\omega}-\Delta\boldsymbol{L}_{\vec{q}}(\vec{Q})+\Delta\mathbf{\Sigma}^{\omega}_{\vec{q}}\right)\mathbf{\Phi}^{\omega}_{\vec{q}}(\vec{Q},\Omega)$$
$$= \Delta\boldsymbol{G}^{\omega}_{\vec{q}}\left[\ \ \otimes\ \ + \int\frac{\mathrm{d}q''}{(2\pi)^3}\boldsymbol{\gamma}^{\omega}_{\vec{q},\vec{q}\,''}\mathbf{\Phi}^{\omega}_{\vec{q}\,''}\right], \qquad (7)$$

with the shorthand notations

$$\Delta\boldsymbol{g}_{\omega} = \left(\omega+\frac{\Omega}{2}\right)^2\boldsymbol{\varepsilon}_b\otimes\ \ -\left(\omega-\frac{\Omega}{2}\right)^2\ \ \otimes\boldsymbol{\varepsilon}_b^{*} \qquad (8)$$

$$\Delta\boldsymbol{L}_{\vec{q}}(\vec{Q}) = \boldsymbol{L}_{\vec{q}_+}\otimes\ \ -\ \ \otimes\boldsymbol{L}_{\vec{q}_-} \qquad (9)$$

$$\Delta\mathbf{\Sigma}^{\omega}_{\vec{q}} = \mathbf{\Sigma}^{\omega_+}_{\vec{q}_+}\otimes\ \ -\ \ \otimes\left(\mathbf{\Sigma}^{\omega_-}_{\vec{q}_-}\right)^{*} \qquad (10)$$

$$\Delta\boldsymbol{G}^{\omega}_{\vec{q}} = \boldsymbol{G}^{\omega_+}_{\vec{q}_+}\otimes\ \ -\ \ \otimes\left(\boldsymbol{G}^{\omega_-}_{\vec{q}_-}\right)^{*}. \qquad (11)$$

The solution of the transport equation, Eq. (7), is found with the help of a momnet expansion of the correlatioon tensor $\mathbf{\Phi}$ itself, which is analogous to the so-called P_1 approximation of the standard Boltzmann equation [8]. As final result of this solution scheme, we find e.g. the energy density correlation tensor displays the expected diffusion pole structure. From this structure, the diffusion coefficient may be extracted, which reads for instance for the bare diffusion

$$\boldsymbol{D}_0 = \frac{1}{3}\boldsymbol{v}_E\boldsymbol{\ell}_T \qquad (12)$$

with the energy transport velocity $\boldsymbol{v}_E = 2c^2\boldsymbol{\varepsilon}_b^{-1}\boldsymbol{c}_p^{-1}[\ \ \otimes\ \ +\boldsymbol{\Delta}(\omega)]$ and transport mean free path $\boldsymbol{\ell}_T = 6i\boldsymbol{c}_p\tilde{\boldsymbol{\ell}}_0/\omega$.

CONCLUSION

In conlcusion, we have presented a microscopic theory of light propagation for disordered anisotropic media including localization effects, based on a self-consistent resummation of cooperon, i.e. self-interference, contributions. The vector character of electromagnetic waves is taken into account as well as dielectric anisotropy by means of a non-trivial dielectric tensor. Energy conservation is incorporated by means of a Ward identity and expressions for transport quantities, such as the diffusion coefficient, are derived.

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