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# Theoretical Approach to Random Lasing in thin Systems on reflecting Substrates

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**Abstract.**
We develop a theory for random laser systems especially for different experimentally relevant setups [1]. A systematical semi-analytical transport theory for amplifying random media is presented. The optical gain is described self-consistently by coupling the transport theory to the semiclassical rate equations and solving this system. In order to stabilize a stationary lasing mode, we include necessarily a loss of photons through the surface of the sample. In experiments [1,2] the disordered laser active material is placed on a substrate and then optically pumped. The free surface on the one side of the sample and the substrate on the other side of the sample yield different conditions for the lasing mode, which we include by considering asymmetric boundary conditions by means of loss through the free surface and reflection on the substrate side, respectively. We calculate all relevant quantities such as the mean photon number and the correlation length for these systems, and compare these results with symmetric systems. In particular the derived correlation lengths in both cases can experimentally be measured as the average spot size of the lasing modes, the spatially confined peaks of the lasing intensity at the surface.

**Keywords:** Random Lasing, Light Propagation in Disordered Media, Weak Localization
**PACS:** 42.25.Bs, 42.25.Dd, 42.25.Hz, 42.55.Zz, 72.15.Rn, 73.20.Fz, 78.20.Ci

## INTRODUCTION

Very recently, light propagation in optically amplifying media has re-attracted a lot of interest. Some progress has been obtained in both, experiments [1, 2, 3, 4], and theory [5, 6, 7]. On theory side one finds works that only consider linear gain only or methods that incorporate dynamical gain behavior by purely numerical approaches. We present a systematical theory for the interplay of strong localization effects and absorption or gain of classical waves in three-dimensional, disordered dielectrics, which is based on a self-consistent resummation of self-interference (Cooperon) contributions. In the presence of absorption or gain, we find that Anderson localized modes do not exist in a strict sense. However the system exhibits a finite correlation length, even though the mean photon number itself is homogeneous. The diffusing photons inside the laser active medium cause stimulated emission which we model by rate equations for a four level laser. By combining the laser rate equations with the transport theory, we describe the random lasing process self-consistently. Size-dependent effects are incorporated.

## MODEL

Here we discuss differerent samples, systems which are of limited size in the third direction. We incorporate some of the material properties of the recent experiments such as asymmetric surface terms. Diffusive loss through the surface and a finite reflection coefficient at the boundary of the system is included and implemented for symmetric and asymmetric conditions. They refer to the experimental situation, where the amplifying Random Lasing material is limited by a surface coupled to air on the one side and by an absorbing material on the other side. This is due to the fact that in the experiment the sample shows a short absorption length which leads to an almost two-dimensional situation. The material of the laser active film is a random medium and often consists of powder of ZnO particles of the size of the incident wavelength.

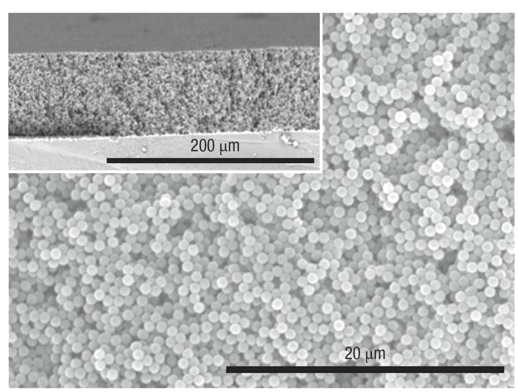


**FIGURE 1.** Experimental setup of a ZnO layer sample on an absorbing substrate. [3]

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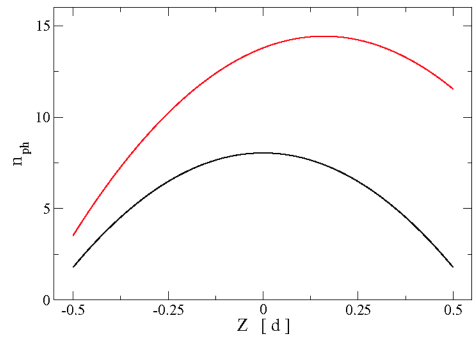


**FIGURE 2.** Comparison of the mean photon number in dependence of the sample depth in a system with symmetric or asymmetric surface conditions.

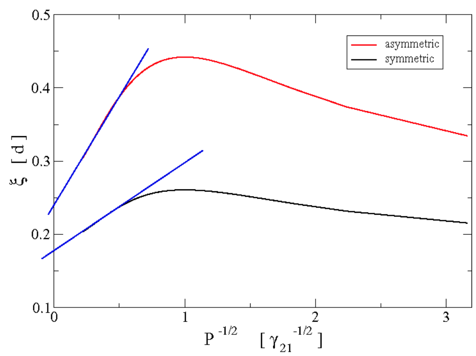


**FIGURE 3.** The correlation length is shown for samples with symmetric and asymmetric boundary conditions. We find a squareroot-behavior well above the threshold which agrees with the results for the spot size in the experiments by H. Cao [4]

In our approach we combine a self-consistent microscopic transport theory with the laser rate equations in the stationary limit. Through this procedure we obtain the quantities discribing the Random Laser. In particular we calculated the mean photon number as a function of the pumping rate and the depth in the sample Fig.2 and compare symmetric to asymmetric case.

We also show the results for the self-consistently calculated correlation length in Fig. 2 which corresponds to the spotsize. We compare the spotsizes for both cases in dependece of the $P^{-1/2}$ which shows us the squareroot behavior well above the threshold for symmmetric and asymmetric boundary conditions.

## CONCLUSION

In conclusion we present a semianalytical, self-consistent theory for thin Random Lasers under different experimental conditions. The Random Laser itself consists of laser active particles, which we treat as identical spheres. These individual scatterers consist for instance of ZnO, as used in various experiments. The width of such films may vary from some ten microns up to two hundred mirometer or more. Our theory is valid for all systems where the width of the film is larger than the wavelength of the light, which is often the case in experiments. By combining the diffusingly transported energy density in the system with the stationary solution of the laser rate equations, we solve this system of equations. Therefore the solutions include quantities such as population inversion of the electronic levels of the lasing material, the average photon number or density, and also the the correlation length of the diffusing lasing modes as the outstanding quantities of the system. We find in particular that the correlation length of the modes determines the spatial size of the the lasing spots, and represents therefore an experimentally accessible quantity. The calculated spot size agrees well with measured spot size in experiments [4].

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